## 7.8 – More About Finding Solutions

*Daily Objectives:*

1. Find the imaginary roots of a polynomial using the real roots.

> **Factor Theorem**
>
> $(x - r)$ is a factor of a polynomial function $P(x)$ if and only if $P(r) = 0$.

**Example 1:** Given the polynomial, $P(x) = x^5 - 6x^4 + 20x^3 - 60x^2 + 99x - 54$:

a. How many roots should this polynomial have? 5

b. What are the real roots of the polynomial?

$x = 1 \quad x = 2 \quad x = 3$

$P(1) = 1^5 - 6(1)^4 + 20(1)^3 - 60(1)^2 + 99(1) - 54 = 0$

$P(2) = 2^5 - 6(2)^4 + 20(2)^3 - 60(2)^2 + 99(2) - 54 = 0$

$P(3) = 3^5 - 6(3)^4 + 20(3)^3 - 60(3)^2 + 99(3) - 54 = 0$

c. What does that mean about the remaining roots?

They are imaginary

d. Find the remaining roots:

**Synthetic Division**

$(x^5 - 6x^4 + 20x^3 - 60x^2 + 99x - 54) \div (x - 1)$

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 1 | -6 | 20 | -60 | 99 | -54 |
| | | 1 | -5 | 15 | -45 | 54 |
| | 1 | -5 | 15 | -45 | 54 | 0 |

$(x^4 - 5x^3 + 15x^2 - 45x + 54) \div (x - 2)$

| | | | | | |
|---|---|---|---|---|---|
| 2 | 1 | -5 | 15 | -45 | 54 |
| | | 2 | -6 | 18 | -54 |
| | 1 | -3 | 9 | -27 | 0 |

$(x^3 - 3x^2 + 9x - 27) \div (x - 3)$

| | | | | |
|---|---|---|---|---|
| 3 | 1 | -3 | 9 | -27 |
| | | 3 | 0 | 27 |
| | 1 | 0 | 9 | |

$\boxed{x^2 + 9}$

$x^2 + 9 = 0$

$-9 \quad -9$

$x^2 = -9$

$\sqrt{x^2} = \sqrt{-9}$

$x = \pm\sqrt{-1 \cdot 9}$

$x = \pm\sqrt{-1}\sqrt{9}$

$\boxed{x = \pm 3i}$ ← Remaining Roots

$\boxed{P(x) = (x-1)(x-2)(x-3)(x^2+9)}$

↑ Factored Form

## Rational Root Theorem

If the polynomial equation $P(x) = 0$ has rational roots, they are in the form $\frac{p}{q}$, where $p$ is a factor of the constant term and $q$ is a factor of the leading coefficient.

Possible Roots : $\pm\frac{1}{1}$ $\pm\frac{1}{3}$ $\pm\frac{5}{1}$ $\pm\frac{5}{3}$ $\pm\frac{25}{1}$ $\pm\frac{25}{3}$ $\Rightarrow \pm 1, \pm\frac{1}{3}, \pm 5 \pm 1\frac{2}{3}, \pm 25 \pm 8\frac{1}{3}$

**Example 2:** Find the roots of this polynomial equation: $\overset{q}{3}x^3 + 5x^2 - 15x - \overset{p}{25} = 0$

**Synthetic Division:**

$P(-\frac{5}{3}) = 3(-\frac{5}{3})^3 + 5(-\frac{5}{3})^2 - 15(-\frac{5}{3}) - 25 = 0 \Rightarrow (x+\frac{5}{3})$ is a factor

$-\frac{5}{3} \cdot -\frac{15}{1} = 25$

$$\begin{array}{r|rrrr} -\frac{5}{3} & 3 & 5 & -15 & -25 \\ & & -5 & 0 & 25 \\ \hline & 3 & 0 & -15 & 0 \end{array}$$

$3x^2 - 15$ $\Rightarrow (x+\frac{5}{3})(3x^2-15)=0$

ONLY WAY TO GET EXACT ANSWERS

$3x^2 - 15 = 0$

$\frac{3x^2}{3} = \frac{15}{3}$

$x^2 = 5$

$x = \pm\sqrt{5} = \pm 2.24$

$\boxed{x = -1\frac{2}{3} \quad x = \sqrt{5} \quad x = -\sqrt{5}}$

**Example 3:** Divide: $(x^3 + 5x^2 - 18) \div (x + 3)$

$$\begin{array}{r|rrrr} -3 & 1 & 5 & 0 & -18 \\ & & -3 & -6 & 18 \\ \hline & 1 & 2 & -6 & 0 \end{array}$$

$(x+3)(x^2+2x-6) = x^3+5x^2-18$

$\boxed{x^2 + 2x - 6}$